

Minimal Conformal Technicolor and Precision Electroweak Tests

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Abstract

We study the minimal model of conformal technicolor, an $SU(2)$ gauge theory near a strongly coupled conformal fixed point, with conformal symmetry softly broken by technifermion mass terms. Conformal symmetry breaking triggers chiral symmetry breaking in the pattern $SU(4) \rightarrow Sp(4)$, which gives rise to a pseudo-Nambu-Goldstone boson that can act as a composite Higgs boson. The top quark is elementary, and the top and electroweak gauge loop contributions to the Higgs mass are cut off entirely by Higgs compositeness. In particular, the model requires no top partners and no “little Higgs” mechanism. A nontrivial vacuum alignment results from the interplay of the top loop and technifermion mass terms. The composite Higgs mass is completely determined by the top loop, in the sense that m_h/m_t is independent of the vacuum alignment and is computable by a strong-coupling calculation. There is an additional composite pseudoscalar A with mass larger than m_h and suppressed direct production at LHC. We discuss the electroweak fit in this model in detail. Corrections to $Z \rightarrow \bar{b}b$ and the T parameter from the top sector are suppressed by the enhanced $Sp(4)$ custodial symmetry. Even assuming that the strong contribution to the S parameter is positive and usuppressed, a good electroweak fit can be obtained for $v/f \lesssim 0.25$, where v and f are the electroweak and chiral symmetry breaking scales respectively. This requires fine tuning at the 10% level.

1 Introduction

At the threshold of the LHC era, the origin of electroweak symmetry breaking remains mysterious. The two major experimental hints we have are the negative results of searches for Higgs bosons, and the constraints from precision electroweak measurements. Known models that explain both of these facts have either residual fine tuning or complicated structure (or both). Electroweak symmetry breaking by strong dynamics is a theoretically compelling paradigm that naturally explains the absence of a standard model Higgs boson, but generally has difficulties explaining the precision electroweak data. The other major problem with strong electroweak symmetry breaking is explaining the quark and lepton masses without flavor-changing neutral currents.

Conformal technicolor [1] is a plausible paradigm for addressing the flavor problem of strong electroweak symmetry breaking. The basic idea is that the strong sector is near a conformal fixed point at high energy scales, with conformal invariance explicitly but softly broken, for example by fermion mass terms. Conformal symmetry breaking then triggers electroweak symmetry breaking. This is a natural solution to the hierarchy problem in models where the conformal breaking terms can be naturally small due to symmetries, as for fermion mass terms. This can help with the flavor problem because in a general conformal fixed point the Higgs operator \mathcal{H} whose VEV breaks electroweak symmetry can have any dimension $d \geq 1$. For $d = 1$ the operators that generate quark and lepton masses scale like standard model Yukawa couplings, and flavor can be decoupled to arbitrarily high scales. However, for $d \rightarrow 1$ the correlation functions of \mathcal{H} approach those of a free field, and we recover the usual hierarchy problem. This occurs because the dimension of $\mathcal{H}^\dagger \mathcal{H}$ approaches $2d \rightarrow 2$ in this limit. The idea is that $d = 1 + 1/\text{few}$, so that large anomalous dimensions allow $\mathcal{H}^\dagger \mathcal{H}$ to be irrelevant, while still allowing the flavor problem to be pushed to high scales.

In this paper, we consider the minimal model of conformal technicolor, $SU(2)$ gauge theory with 2 electroweak doublets with addition electroweak-singlet technifermions so that the theory has a strongly-coupled conformal fixed point [2]. Mass terms for technifermions softly break the conformal symmetry and give rise to electroweak symmetry breaking at low scales. This model naturally has a composite Higgs boson, and we will show that a good precision electroweak fit can be obtained with very mild fine tuning.

The effective theory below the chiral symmetry breaking scale is determined by the symmetry breaking pattern $SU(4) \rightarrow Sp(4)$ (equivalent to $SO(6) \rightarrow SO(5)$), which gives rise to 5 potential Nambu-Goldstone bosons. Of these, 3 become the

longitudinal components of the W and Z gauge boson. The remaining 2 physical pseudo-Nambu-Goldstone bosons (PNGBs) are a composite Higgs scalar h and a pseudoscalar A . The theory has a nontrivial vacuum alignment parameterized by an angle θ defined by

$$v = f \sin \theta, \quad (1.1)$$

where $v = 246$ GeV is the electroweak vacuum expectation value (VEV) and f is the decay constant of the Nambu-Goldstone bosons. Here $\sin \theta = 1$ corresponds to the “technicolor” vacuum where electroweak symmetry is broken by the strong dynamics, while for $\sin \theta \ll 1$ electroweak symmetry is broken by the VEV of the composite Higgs. The dominant contributions to the potential that determines θ naturally come from top loops and from explicit technifermion masses. The top loop completely determines the h mass, in the sense that

$$m_h^2 = c_t N_c m_t^2, \quad (1.2)$$

where $N_c = 3$ is a color factor and $c_t \sim 1$ is an effective coupling in the low-energy effective theory that parameterizes how the Higgs compositeness cuts off the top loop. The mass is independent of θ , but it is only for small θ that h couples to electroweak gauge bosons like the standard model Higgs. The other PNGB is the pseudoscalar A , which has mass

$$m_A^2 = \frac{m_h^2}{\sin^2 \theta}. \quad (1.3)$$

The pseudoscalar is therefore heavier than the scalar. For $m_A > 2m_t$ the A decays dominantly to $t\bar{t}$, but for $150 \text{ GeV} \lesssim m_A < 2m_t$ it decays dominantly to WW , ZZ , and $Z\gamma$ (but not $\gamma\gamma$). However, the A couplings to electroweak gauge bosons and tops are strongly suppressed compared to the analogous Higgs couplings, and direct A production is negligible at the LHC. It is possible that TeV resonances decaying to A particles may give an observable cross section, but we leave the investigation of this to future work.

We investigate the precision electroweak constraints on this model in some detail. It is often stated that models of strong electroweak symmetry breaking are strongly disfavored by precision electroweak data. The situation is illustrated in Fig. 1, which shows the expectations for the S and T parameters in a theory that is strongly coupled at the TeV scale, with no light Higgs boson and no custodial symmetry breaking other than the top mass and the gauging of $U(1)_Y$. We also assume that the S and T parameters are not enhanced by large N factors. The present model with

$\sin \theta = 1$ satisfies all these conditions. A good electroweak fit can be obtained with an additional positive contribution to T and a small or negative S parameter.

Is it plausible that $S < 0$ in the present model? Experimental data can be used to show that $S > 0$ in strong electroweak sectors that are scaled-up versions of QCD [3]. However, it is noteworthy there is no theoretical argument that $S > 0$ for vectorlike gauge theories despite the fact that many similar inequalities have been proven. In gauge theories like QCD or the present theory, S can be expressed in terms of the physical states of the theory, and is dominated by states with energies near the strong coupling scale Λ . Conformal theories such as the one we are discussing are very different from QCD above the scale Λ , so there is no reason to expect the states near the scale Λ to be well-modeled by QCD. It is therefore quite possible that $S < 0$ in this theory, and the technicolor vacuum has a perfectly good electroweak fit. The fact that strong electroweak symmetry breaking may have a good electroweak fit is much more general than the present model, and should be taken seriously when assessing the plausibility of strong electroweak symmetry breaking signals at the LHC.

Nonetheless, there are reasons to suspect that $S > 0$ in strongly-coupled theories such as the one under discussion. In every known theory in which S has a calculable sign, it turns out to be positive. For example, simple extensions of the standard model such as extra $SU(2)_W$ doublets give a positive contribution to S . More relevant for the present discussion are 5D “Higgless” models of electroweak symmetry breaking [5]. These can be viewed as “dual” descriptions of large- N strongly-coupled electroweak symmetry breaking, and are conformal if the 5D spacetime is AdS. These models also predict $S > 0$ when S is calculable [6].

This motivates us to investigate whether there can be a good precision electroweak fit if the contribution to S from the physics above the scale Λ is positive and unsuppressed and unsuppressed, *e.g.* in the “QCD” region of Fig. 1. In the present model, away from the technicolor vacuum where $\sin \theta = 1$ there are additional negative contributions to the S parameter from loops of the composite Higgs boson. These also give a positive contribution to the T parameter, which further improves the electroweak fit. These contributions go in the right direction because the theory becomes a (finely-tuned) standard model for $\theta \ll 1$ with a light composite Higgs boson. We will find that $\theta \lesssim 0.25$ is sufficient to get a good electroweak fit in the present model, requiring fine tuning of order $\sin^2 \theta \sim 10\%$. It is also possible that there are additional contributions to the T parameter that allow even larger values of $\sin \theta$ (and less fine-tuning), but we confine ourselves to the minimal model in this paper.

Composite Higgs models were first introduced in the context of strongly-coupled gauge theories in the 1980’s [7] and more recently revived in the context of 5D models

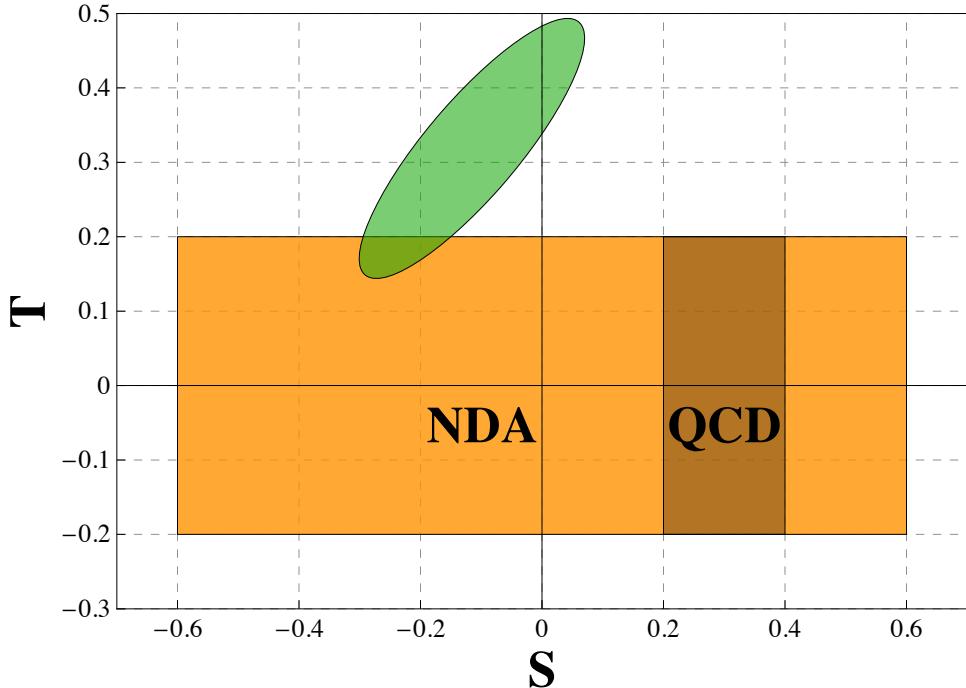


Fig. 1. Expected range of S and T parameters in a general theory of electroweak symmetry breaking that is strongly coupled at the TeV scale. The reference Higgs mass is taken to be 1 TeV. The region denoted by NDA (“naïve dimensional analysis”) is what is expected in a general theory of strong electroweak symmetry breaking [4]. The region denoted by QCD is what is expected in a theory of scaled-up QCD.

[8]. The present model is similar in spirit to the early composite Higgs models, but it is based on a conformal rather than an asymptotically free gauge theory. The large coupling to the top quark is another important new ingredient in the present model. Asymptotically free $SU(2)$ gauge theories that give rise to the symmetry breaking pattern $SU(4) \rightarrow Sp(4)$ were considered as composite Higgs theories in the second paper in Ref. [7]. Ref. [9] analyzes a version of this theory where the top quark is included and top partners are introduced to raise the scale of compositeness above the TeV scale. Ref. [10] analyzes a 5D model with the same coset, but considers a different stabilizing potential with different phenomenology. In the 5D models, the top loop contribution to the Higgs mass are also off by top partners. In the present model, the top quark contribution to the composite Higgs mass is cut off entirely by compositeness of the Higgs sector, and there is strong dynamics near the TeV scale. The experimental signature of the top quark coupling to the symmetry breaking sector

is the presence of strong spin-0 resonances coupling to the top quark [11].

Another important difference between the present model and the 5D composite models is that the latter are effectively $1/N$ expansions of a “dual” strongly-coupled large- N theory. In the 5D description, N counts the number of KK modes below the UV cutoff of the theory, and hence parameterizes the range of validity of the 5D effective theory. These theories have a calculable positive contribution to the S parameter proportional to N , and this contribution must be canceled by fine-tuning. This means that 5D composite Higgs theories are inevitably a compromise between fine-tuning and predictivity of the effective theory. This compromise is absent in the present theory, which has a definite UV completion in terms of an $SU(2)$ gauge theory, and has no large- N enhancement of the S parameter. The UV completion is strongly-coupled, but may be investigated on the lattice or using tools of conformal field theory. Both are under active investigation and may give nontrivial constraints on this class of models in the near future.

This paper is organized as follows. In Section 2, we present the model and review the constraints on the dimension of the Higgs operator. In Section 3, we determine the vacuum alignment, find the spectrum and interactions of the PNGBs, and discuss the collider phenomenology of the model. In Section 4, we study the precision electroweak constraints in this model. Section 5 contains our conclusions. Some technical results are collected in Appendices.

2 The Model

2.1 Minimal Conformal Technicolor

We begin by defining the model [2]. It has a new strong $SU(2)$ gauge group, with fermions transforming under $SU(2)_{\text{CTC}} \times SU(2)_W \times U(1)_Y$ as

$$\begin{aligned} \psi &\sim (2, 2)_0, \\ \tilde{\psi}_1 &\sim (2, 1)_{-\frac{1}{2}}, \\ \tilde{\psi}_2 &\sim (2, 1)_{+\frac{1}{2}}, \\ \chi &\sim (2, 1)_0 \times 2n. \end{aligned} \tag{2.1}$$

The fields ψ and $\tilde{\psi}$ have the quantum numbers of minimal technicolor [12], while χ are technifermions with no standard model charges. There are $2n$ copies of χ fermions, enough so that the theory has a strongly-coupled conformal fixed point. There is a growing body of lattice evidence for the existence of such a fixed point in $SU(3)$

gauge theory near 12 flavors, although important disagreements between different groups remain [13]. There are currently no relevant lattice simulations for $SU(2)$ gauge theory, but comparison with supersymmetric theories leads one to expect a strong conformal fixed point for $n \simeq 4$ [14].

The theory also contains mass terms for the technifermions:

$$\Delta\mathcal{L} = -\kappa\psi\psi - \tilde{\kappa}\tilde{\psi}_1\tilde{\psi}_2 - K\chi\chi + \text{h.c.} \quad (2.2)$$

Although we refer to these as mass terms, the fermion bilinears have a nontrivial scaling dimension $1 < d < 3$, so κ , $\tilde{\kappa}$, and K have dimension $4 - d$. They are relevant perturbations, and they take the theory out of the conformal fixed point at the scale where they become strong. We assume $\kappa, \tilde{\kappa} \ll K$ so that conformal breaking is dominated by K . The scale of confinement and conformal symmetry breaking is then

$$\Lambda \sim K^{1/(4-d)}. \quad (2.3)$$

At the scale Λ , we assume that the theory is in the same universality class as $SU(2)$ gauge theory with 4 fundamentals. This can be motivated by extrapolating from a model where the fixed point is weakly coupled, for example in the theory with larger n near the Banks-Zaks fixed point [15]. In this case, the theory is weakly coupled at the scale K , and we can integrate out the massive technifermions perturbatively. The effective theory below this scale is an $SU(2)$ gauge theory with 4 fundamentals, which is asymptotically free and becomes strongly coupled at a scale below K . It is believed to confine and break chiral symmetry in analogy with QCD. We assume that the picture is qualitatively similar in the strongly coupled case, with the important difference that the coupling is already strong at the scale K .

The theory has an approximate $SU(4)$ symmetry under which the light fermions rotate into each other. We therefore define a 4-component fermion vector

$$\Psi = \begin{pmatrix} \psi \\ \tilde{\psi}_1 \\ \tilde{\psi}_2 \end{pmatrix}. \quad (2.4)$$

The condensate is

$$\langle \Psi^a \Psi^b \rangle \propto \Phi^{ab} = -\Phi^{ba} \quad (2.5)$$

with a constant of proportionality of order Λ^d . We assume that Φ has maximal rank, and has maximal symmetry, in which case we have the symmetry breaking pattern $SU(4) \rightarrow Sp(4)$. This naturally generates electroweak symmetry breaking via composite Higgs boson as we discuss below.

The scale Λ is put in “by hand” via the mass term K in Eq. (2.2). This is an explicit but soft breaking of the conformal symmetry, so it is natural for K to be small compared to fundamental scales such as the Planck scale. This gives a completely natural solution of the hierarchy problem that is conceptually similar to softly broken supersymmetry. (In fact, both conformal symmetry and supersymmetry are nontrivial extensions of Lorentz invariance, so both involve a new broken spacetime symmetry.) Soft supersymmetry breaking may arise from spontaneous symmetry breaking in a more fundamental theory, explaining the origin of the soft breaking terms. Similarly, the mass terms in Eq. (2.2) can arise from a variety of different ways, including spontaneous breaking of chiral symmetries, gauged or not. We will not discuss the details here, and instead focus on the phenomenology of the model at the TeV scale.

2.2 Top Quark Mass

The main phenomenological virtue of strong conformal dynamics above the TeV scale is that it allows a plausible theory of flavor, especially the origin of the top quark mass [1]. The top quark mass arises from a higher-dimension operator of the form

$$\Delta\mathcal{L}_{\text{top}} = \frac{g_t^2}{\Lambda_t^{d-1}} (Qt^c)^\dagger (\psi\tilde{\psi}_1) + \text{h.c.}, \quad (2.6)$$

where d is the scaling dimension of $\psi\tilde{\psi}$ (the same as that of $\chi\chi$ since the theory has a $SU(2n+4)$ symmetry at the conformal fixed point) and g_t is a dimensionless coupling. The low-energy physics depends only on the ratio g_t^2/Λ_t^{d-1} , and it is convenient to choose g_t so that Λ_t is the scale where the operator gets strong. The top mass is then given by

$$m_t \sim \Lambda \left(\frac{\Lambda}{\Lambda_t} \right)^{d-1} \sin \theta, \quad (2.7)$$

where the θ dependence arises because electroweak symmetry is unbroken in the limit $\theta \rightarrow 0$. Using

$$\Lambda \sim 4\pi f \sim \frac{4\pi v}{\sin \theta}, \quad (2.8)$$

we have

$$\Lambda_t \sim \frac{4\pi v}{\sin \theta} \left(\frac{4\pi v}{m_t} \right)^{1/(d-1)}. \quad (2.9)$$

Using $\sin \theta \sim 0.25$ and $4\pi v \sim 2$ TeV, $m_t \simeq 165$ GeV (the “Yukawa mass”), we obtain

$$\Lambda_t \sim \begin{cases} 30 \text{ TeV} & d = 3, \\ 100 \text{ TeV} & d = 2, \\ 1000 \text{ TeV} & d = 1.5. \end{cases} \quad (2.10)$$

For the free field theory value $d = 3$, $\Lambda_t \sim 3\Lambda$, and the dynamics that generates the top mass cannot be disentangled from that of electroweak symmetry breaking. This possibility is explored in “topcolor” models [16]. However, for $d < 3$ there can be a large separation of scales. This is the basic idea of “walking” technicolor, which goes back to the 1980’s [17].

2.3 Bounds on Dimensions

How small must d be in order to have a sensible theory of flavor? The answer depends on what flavor symmetries (if any) are preserved by the flavor dynamics. The minimum requirement is that flavor dynamics occurs at a scale at or below Λ_t , the scale where the top quark coupling Eq. (2.6) gets strong. If the flavor dynamics satisfies minimal flavor violation, then the scale of flavor physics may be quite low, and we do not require very small values of d . This may happen if the couplings of the standard model fermions to the strong dynamics arises from the exchange of a heavy scalar doublet with vanishing VEV. Such a scalar may be natural in theories where supersymmetry is broken at high scales, as in “bosonic technicolor” [18] (see also [19]). In such a theory, the only flavor structure comes from the scalar couplings to standard model fermions, so minimal flavor violation is automatic.

If the theory of flavor does not have natural flavor conservation, but only Yukawa suppression of flavor violation, then we expect effective flavor-violating operators of the form

$$\Delta\mathcal{L}_{\text{eff}} \sim \frac{y_d y_s}{\Lambda_f^2} (\bar{s}d)^2 + \dots \quad (2.11)$$

The strongest bound on these operators comes from the CP -violating part of $K^0 - \bar{K}^0$ mixing, which gives

$$\Lambda_f \gtrsim 30 \text{ TeV}. \quad (2.12)$$

The top quark coupling gets strong near 30 TeV for $d = 3$ (the free field value), so a strongly coupled theory of flavor in principle may not require conformal dynamics. Realistic theories of flavor probably require weak coupling, in which case the minimal requirement is $d < 3$. We leave detailed discussion of flavor models to future work, but our point is that we do not necessarily need very small values of d to get a plausible theory of flavor.

We now discuss theoretical constraints on the value of d . One important tool is lattice studies. The value of d in strongly coupled gauge theories can be measured

by a lattice calculation using finite-size scaling techniques [2]. These calculations are now starting to become a reality [20].

General results from conformal field theory also restrict the value of d . Unitarity implies $d \geq 1$, and in the limit $d \rightarrow 1$ the correlation functions of the operator becomes those of a free scalar [21]. This means that sufficiently close to $d = 1$ the “Higgs” operator $\mathcal{H} = \psi\tilde{\psi}$ becomes a weakly-coupled scalar field, and we get back the hierarchy problem of the standard model. This can be stated in conformal field theory language as follows [1]. The operator product expansion of \mathcal{H} with itself contains operators

$$\mathcal{H}^a(x)\mathcal{H}_b^\dagger(0) \sim \delta_b^a \mathbf{1} + \delta_b^a [\mathcal{H}^\dagger\mathcal{H}](0) + (\sigma_i)_b^a [\mathcal{H}^\dagger\sigma_i\mathcal{H}](0) + \dots \quad (2.13)$$

The “Higgs mass term” $[\mathcal{H}^\dagger\mathcal{H}]$ is a singlet under all symmetries, and therefore cannot be forbidden from appearing in the effective Lagrangian. To have a stable fixed point and avoid the hierarchy problem, we therefore require that it is an irrelevant operator:

$$\Delta = \dim[\mathcal{H}^\dagger\mathcal{H}] > 4. \quad (2.14)$$

In the weakly-coupled limit $d \rightarrow 1$, we have $\Delta \rightarrow 2$, and we recover the usual hierarchy problem. The $d \rightarrow 1$ limit was investigated in detail in Ref. [22]. They found that as $d \rightarrow 1$

$$\Delta \leq 2d + \mathcal{O}((d-1)^{1/2}), \quad (2.15)$$

so the limit is approached rather slowly. The authors also derived quantitative limits on the quantity

$$\Delta_{\min} = \min\{\dim[\mathcal{H}^\dagger\mathcal{H}], \dim[\mathcal{H}^\dagger\sigma_i\mathcal{H}]\}. \quad (2.16)$$

The techniques used do not distinguish operators that differ only by internal symmetries, so they are not able to bound Δ . The limits on Δ_{\min} are quite restrictive [22, 23], but it is important to keep in mind that these bounds do not apply to the quantity of interest for these models.

3 Vacuum Alignment

We now discuss the vacuum structure and electroweak symmetry breaking in this model. In the basis Eq. (2.4) the $SU(2)_L \times SU(2)_R$ generators are

$$T = \begin{pmatrix} t_L & 0 \\ 0 & -t_R^T \end{pmatrix}, \quad (3.1)$$

where $SU(2)_L = SU(2)_W$ and $Y = T_{3R}$. The condensate Eq. (2.5) then breaks the $SU(4)$ global symmetry down to $Sp(4)$. Electroweak breaking depends on the alignment of the vacuum. For example, there are vacua

$$\Phi \propto \begin{pmatrix} \epsilon & 0 \\ 0 & \pm\epsilon \end{pmatrix}, \quad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (3.2)$$

where $SU(2)_L \times SU(2)_R$ is unbroken. We refer to these as “electroweak vacua.” We also have a vacuum where

$$\Phi \propto \begin{pmatrix} 0 & 1_2 \\ -1_2 & 0 \end{pmatrix} \quad (3.3)$$

where electroweak symmetry is maximally broken by the strong dynamics. We refer to this as the “technicolor vacuum.”

We will be interested in vacua between these limits. In Appendix A, it is shown that the most general condensate up to $SU(2)_W$ transformations is given by either Φ or $i\Phi$, where

$$\Phi = \begin{pmatrix} e^{i\alpha} \cos \theta \epsilon & \sin \theta 1_2 \\ -\sin \theta 1_2 & -e^{-i\alpha} \cos \theta \epsilon \end{pmatrix}, \quad (3.4)$$

with $0 \leq \theta \leq \pi$. Note that the phase of the condensate is meaningful once we require that the constant of proportionality in Eq. (2.5) is real. The condensates Φ and $i\Phi$ are not related by any symmetry of the $SU(2)$ gauge theory, and are therefore physically distinct. (In particular, they are not associated with degenerate vacua and domain walls.) Our final results do not depend on which choice represents the physical vacua, and we will use Φ .

The coset $SU(4)/Sp(4)$ contains 5 generators, 3 of which correspond to the longitudinal polarizations of the massive W and Z . The remaining 2 generators are physical PNGBs, which are here parameterized by θ and α . We will be interested in CP conserving vacua where $\alpha = 0$.

The electroweak gauge boson masses arise from the covariant kinetic term for the PNGBs:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} f^2 \text{tr}(\Omega^{\perp\mu} \Omega_\mu^\perp) = \frac{1}{8} g^2 f^2 \sin^2 \theta W^{+\mu} W_\mu^- + \dots \quad (3.5)$$

(The formalism and notation is described in the Appendix.) From this we see that the electroweak breaking scale is

$$v = f \sin \theta. \quad (3.6)$$

The value of θ is determined by interactions that explicitly break $SU(4)$. The largest effects are top loops, electroweak gauge boson loops, and the explicit technifermion mass terms κ and $\tilde{\kappa}$ in Eq. (2.2). These generate a potential for the PNGBs, which can also be viewed as a potential for θ . We now discuss these contributions in turn.

3.1 Technifermion Mass

The technifermion mass terms Eq. (2.2) break $SU(4)$ and therefore contribute to the PNGB potential. We are assuming that $K > \kappa, \tilde{\kappa}$ so that K triggers chiral symmetry breaking. Note that if the masses K, κ , and $\tilde{\kappa}$ have a common origin, then it is natural for $\kappa, \tilde{\kappa} \lesssim K$. It is therefore natural for κ and $\tilde{\kappa}$ to be important perturbations at the scale Λ . Note that this relies on the conformal technicolor mechanism for generating the scale Λ . If Λ were a dynamical scale arising from a gauge coupling becoming strong, then there would be no reason for mass terms like κ and $\tilde{\kappa}$ to be important perturbations at the scale Λ .

To keep track of the $SU(4)$ symmetry, we write the small technifermion mass terms as

$$\mathcal{L} = -\Psi^T \mathcal{K} \Psi + \text{h.c.}, \quad (3.7)$$

where (in the basis Eq. (2.4))

$$\mathcal{K} = \begin{pmatrix} \kappa\epsilon & 0 \\ 0 & \tilde{\kappa}\epsilon \end{pmatrix}. \quad (3.8)$$

We then view \mathcal{K} as a spurion transforming under $SU(4)$ as $\mathcal{K} \mapsto U^* \mathcal{K} U^\dagger$. Treating \mathcal{K} as a perturbation, the leading term in the potential in the effective theory below the scale Λ is

$$V_{\text{mass}} = \frac{1}{4} \hat{C}_\kappa \text{tr}(\xi^T \mathcal{K} \xi \Phi) + \text{h.c.} = -\hat{C}_\kappa (\kappa - \tilde{\kappa}) \cos \theta + \mathcal{O}(h, A) \quad (3.9)$$

where ξ parameterizes the PNGB fields (see appendices) and

$$\hat{C}_\kappa \sim \frac{\Lambda^d}{16\pi^2}. \quad (3.10)$$

The estimate Eq. (3.10) can be understood as follows. The scale of the strong dynamics is set by K , and we can normalize the technifermion fields so that $K \sim \Lambda^{4-d}$. Therefore, in the limit $\kappa, \tilde{\kappa} \rightarrow \Lambda^{4-d}$, all the technifermion mass terms are strong at the scale Λ , and the theory has a single scale. In this limit, all contributions to the vacuum energy are expected to be of order $\Lambda^4/16\pi^2$, the same as the vacuum energy

of a free particle with mass Λ . This fixes the coefficient Eq. (3.10). This argument is equivalent to “naïve dimensional analysis” [4].

In fact, the effective coupling \hat{C}_κ in Eq. (3.9) is related to the effective coupling that determines the top mass, since both arise from the VEV of the technifermion bilinear. The relation is

$$m_t = \frac{g_t^2 \hat{C}_\kappa}{4\Lambda_t^{d-1}}. \quad (3.11)$$

where g_t is the coefficient defined in Eq. (2.6).

3.2 Top Loop

The top quark coupling Eq. (2.6) also breaks $SU(4)$, and top loops give an important contribution to the PNGB potential. To keep track of the $SU(4)$ symmetry, we write the top quark coupling Eq. (2.6) as

$$\Delta\mathcal{L}_{\text{eff}} = \frac{g_t^2}{\Lambda_t^{d-1}} (Qt^c)_\alpha^\dagger \Psi^T P^\alpha \Psi \quad (3.12)$$

where $\alpha = 1, 2$ is a $SU(2)_W$ index and

$$P^1 = \frac{1}{2} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad P^2 = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (3.13)$$

We view P^α as a spurion transforming under $U \in SU(4)$ as $P^\alpha \mapsto U^* P^\alpha U^\dagger$. The leading effective potential induced by top loops is then

$$V_{\text{top}} = -\frac{1}{2} C_t \left| \text{tr}(P^\alpha \xi \Phi \xi^T) \right|^2 = -\frac{1}{2} C_t \sin^2 \theta + \mathcal{O}(h, A), \quad (3.14)$$

where

$$C_t \sim \frac{N_c \Lambda^4}{16\pi^2} \left(\frac{\Lambda}{\Lambda_t} \right)^{2(d-1)}. \quad (3.15)$$

Here $N_c = 3$ is the number of QCD colors. The estimate Eq. (3.15) can be understood by noting that $V_{\text{top}} \sim N_c \Lambda^4 / (16\pi^2)$ when $\Lambda_t \rightarrow \Lambda$, the limit where all interactions are strong at the scale Λ . The sign of C_t depends on the physics at the scale Λ and is not calculable. We assume that $C_t > 0$, which is the sign obtained by computing V_{top} in the low-energy effective theory with a momentum cutoff at the scale Λ . This is also

the sign obtained in models where the top loop is cut off in a calculable way by top partners [25]. The top loop then prefers to break electroweak symmetry.

The top quark mass depends on $\langle\theta\rangle$, the value of θ at the minimum of the potential. In order to keep the physical value of the top quark mass fixed, the coupling of the top to the strong sector must depend on $\langle\theta\rangle$. To work this out, note that the top mass is contained in the term

$$\Delta\mathcal{L}_{\text{eff}} = a_t \Lambda \left(\frac{\Lambda}{\Lambda_t}\right)^{d-1} (Qt^c)_\alpha^\dagger \text{tr}(P^\alpha \xi \Phi \xi^T) + \text{h.c.} \quad (3.16)$$

with $a_t \sim 1$ (so that $m_t \sim \Lambda$ in the limit $\Lambda_t \rightarrow \Lambda$). This gives

$$a_t \Lambda \left(\frac{\Lambda}{\Lambda_t}\right)^{d-1} \sin\langle\theta\rangle = m_t \quad (3.17)$$

and hence

$$C_t \sim \frac{N_c \Lambda^4}{16\pi^2} \left(\frac{\Lambda}{\Lambda_t}\right)^{2(d-1)} \sim \frac{N_c m_t^2 f^2}{\sin^2\langle\theta\rangle}. \quad (3.18)$$

using $\Lambda \sim 4\pi f$.

3.3 Electroweak Gauge Loops

The electroweak gauge couplings also break $SU(4)$ and therefore contribute to the PNGB potential. We view the gauge couplings and generators as spurions transforming as $g_A T_A \mapsto U(g_A T_A) U^\dagger$, where A runs over the gauge bosons and generators. The leading term in the effective potential is then

$$V_{\text{gauge}} = C_g \sum_A g_A^2 \text{tr}(\xi^\dagger T_A \xi \Phi \xi^\dagger T_A \xi \Phi) = \frac{1}{2} C_g (3g^2 + g'^2) \sin^2 \theta + \mathcal{O}(h, A), \quad (3.19)$$

where the sum is over the gauge generators and

$$C_g \sim \frac{\Lambda^4}{(16\pi^2)^2}. \quad (3.20)$$

This estimate can be understood from the fact that $V_g \sim \Lambda^4/16\pi^2$ in the strong coupling limit $g \rightarrow 4\pi$. This contribution was estimated in Refs. [24] using a spectral representation for C_g and assuming that it is saturated by the lowest-lying vector resonances. They obtained

$$C_g \simeq \frac{3m_\rho^2 f^2}{16\pi^2} \quad (3.21)$$

where m_ρ is the mass of the lowest-lying vector resonance. This is consistent with the estimate Eq. (3.20) since $m_\rho \sim \Lambda \sim 4\pi f$. In particular $C_g > 0$, which results from vector meson dominance and the assumption that the lowest-lying axial vector resonance has a mass larger than m_ρ (as in QCD). This favors the electroweak preserving vacuum. However, we have seen that the top quark contribution has the same θ dependence, and plausibly has the opposite sign. These contributions appear to be comparable in size:

$$\frac{C_g(3g^2 + g'^2)}{C_t} \sim \frac{3g^2 + g'^2}{N_c(m_t/v)^2} \lesssim 1. \quad (3.22)$$

In models in which both the top and the gauge loops are cut off in a calculable way, one finds that the top contribution to the Higgs potential dominates [25] and favors the electroweak symmetry breaking vacuum. We will assume that this is the case also in the present model.

3.4 Four-Technifermion Interactions

The dynamics that generates the top quark interaction Eq. (2.6) is expected to also generate four-technifermion interactions such as

$$\Delta \mathcal{L}_{4\psi} \sim \frac{g_{4\psi}^2}{\Lambda_t^{\Delta-4}} |\psi \tilde{\psi}_1|^2 + \dots \quad (3.23)$$

where Δ is the dimension of the 4-fermion operator and $g_{4\psi}$ is a dimensionless coupling that parameterizes the strength of the four-technifermion interaction at the scale Λ_t . In order to have a consistent IR fixed point, we require these terms to be irrelevant, *i.e.* $\Delta > 4$. Estimating the size of these effects compared to top quark loops, we obtain

$$\frac{V_{4\psi}}{V_{\text{top}}} \sim \left(\frac{g_{4\psi}}{4\pi}\right)^2 \left(\frac{m_t}{\Lambda}\right)^{(\Delta-2d-2)/(d-1)}. \quad (3.24)$$

We see that the four-technifermion interactions is suppressed for small $g_{4\psi}$, and is even exponentially suppressed provided that $\Delta > 2d + 2$. This is model-dependent, and we will assume that the top loop dominates in order to work with a minimal model.

3.5 Minimizing the Potential

Collecting the results above, we find that the potential for θ takes the simple form

$$V = -C_\kappa \cos \theta - \frac{1}{2} C_t \sin^2 \theta, \quad (3.25)$$

where

$$C_\kappa = \hat{C}_\kappa(\kappa - \tilde{\kappa}). \quad (3.26)$$

Because we assume $C_t > 0$, the top contribution prefers the “technicolor” vacuum at $\sin \theta = 1$, but the technifermion mass contribution has a tadpole there. The vacuum is therefore generally between the technicolor and electroweak preserving vacuum, exactly what is required to get a composite Higgs boson.

Minimizing the potential, we find extrema at $\theta = 0$ and

$$\cos \theta = \frac{C_\kappa}{C_t}. \quad (3.27)$$

The extremum Eq. (3.27) has lower energy than $\theta = 0$ as long as $|C_\kappa|/C_t < 1$. We choose the technifermion masses so that $C_\kappa > 0$, so that $0 < \theta < \frac{\pi}{2}$.

3.6 PNGB Spectrum

We work out the masses and couplings of the physical PNGBs using a basis of generators where the PNGB fields have vanishing VEVs. The generators then depend on θ (see Appendix B). The generators corresponding to the scalar and pseudoscalar PNGBs are

$$X_h = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & i\epsilon \\ i\epsilon & 0 \end{pmatrix}, \quad (3.28)$$

$$X_A = \frac{1}{2\sqrt{2}} \begin{pmatrix} \cos \theta & -\sin \theta \epsilon \\ \sin \theta \epsilon & -\cos \theta \end{pmatrix}, \quad (3.29)$$

respectively. The masses of the PNGBs can be determined by expanding the potential given above. We find

$$m_h^2 = c_t N_c m_t^2, \quad (3.30)$$

$$m_A^2 = \frac{m_h^2}{\sin^2 \theta}. \quad (3.31)$$

where

$$c_t = \frac{C_t \sin^2 \langle \theta \rangle}{N_c m_t^2 f^2} \sim 1. \quad (3.32)$$

Because of the special form of the potential, m_h/m_t is determined completely by the top loop, and is independent of $\langle \theta \rangle$. This can be understood from the fact that θ can be used to parameterize the Higgs field, so we have

$$m_h^2 = \frac{1}{f^2} \left\langle \frac{\partial^2 V}{\partial \theta^2} \right\rangle = \frac{C_t}{f^2} \sin^2 \langle \theta \rangle = N_c c_t m_t^2. \quad (3.33)$$

This means that the Higgs mass in this model can in principle be determined by a strong-coupling calculation of C_t , which determines the top loop contribution to the vacuum energy. The fact that $m_A \rightarrow m_h$ as $\sin \theta \rightarrow 1$ can be understood from the fact that there is an enhanced $U(1)$ custodial symmetry in this limit, under which h and A form a complex charged multiplet.

3.7 PNGB Interactions

We now discuss the PNGB interactions with standard model particles. The leading couplings of the standard model gauge bosons with the PNGBs can be read off from the kinetic term Eq. (3.5):

$$g_{WWh} = g_{WWh}^{(\text{SM})} \cos \theta, \quad (3.34)$$

$$g_{ZZh} = g_{ZZh}^{(\text{SM})} \cos \theta, \quad (3.35)$$

$$g_{WWhh} = g_{WWhh}^{(\text{SM})} \cos 2\theta, \quad (3.36)$$

$$g_{ZZhh} = g_{ZZhh}^{(\text{SM})} \cos 2\theta, \quad (3.37)$$

$$g_{WWAA} = g^2 \sin^2 \theta, \quad (3.38)$$

$$g_{ZZAA} = (g^2 + g'^2) \sin^2 \theta. \quad (3.39)$$

The 4-point couplings are normalized so that the interaction terms are $\frac{1}{4}g_{ZZAA}A^2Z^\mu Z_\mu$, etc.

The PNGB kinetic term does not give rise to couplings of the form AVV , where $V = W, Z, \gamma$. This is a consequence of CP invariance, under which A is odd (see Appendix C). In order to have a CP invariant AVV coupling we need coupling involving $\epsilon_{\mu\nu\rho\sigma}$. Terms in the effective Lagrangian of the form $\epsilon^{\mu\nu\rho\sigma} \text{tr}(\nabla_\mu \Omega_\nu^\perp \Omega_\rho^\perp \Omega_\sigma^\perp)$ do not contain a AVV vertex, and we must apparently go to higher order in the derivative expansion. In fact, the leading coupling comes from the chiral anomaly. This is exactly analogous to the story for the $\pi^0\gamma\gamma$ coupling in QCD, and we can use the standard derivation based on the chiral anomaly to find these couplings. Writing the couplings as

$$\Delta\mathcal{L}_{\text{eff}} = \frac{1}{4}g_{AV_1V_2}A\epsilon^{\mu\nu\rho\sigma}V_{1\mu\nu}V_{2\rho\sigma} \quad (3.40)$$

we have

$$g_{AWW} = \frac{g^2 \sin \theta \cos \theta}{16\sqrt{2}\pi^2 v}, \quad (3.41)$$

$$g_{AZZ} = \frac{(g^2 - g'^2) \sin \theta \cos \theta}{16\sqrt{2}\pi^2 v}, \quad (3.42)$$

$$g_{AZ\gamma} = \frac{gg' \sin \theta \cos \theta}{16\sqrt{2}\pi^2 v}. \quad (3.43)$$

Interestingly, $g_{A\gamma\gamma}$ vanishes at this order. This is due simply to the vanishing of the trace of charges, so this may be viewed as a consequence of the $SU(4)$ symmetry.

The PNGB fields also have couplings to the top quark via the operator in the effective theory that contains the top quark mass:

$$\Delta\mathcal{L}_{\text{eff}} = \frac{m_t}{2\sin\langle\theta\rangle} (Qt^c)^\dagger_\alpha \text{tr}(\Phi\xi^T P_\alpha \xi) + \text{h.c.} \quad (3.44)$$

There are similar couplings for the remaining standard model fermions, and we find

$$g_{h\bar{f}f} = g_{h\bar{f}f}^{(\text{SM})} \cos \theta. \quad (3.45)$$

There is no contribution to $g_{A\bar{f}f}$ from the couplings above, again due to the $SU(4)$ symmetry. We get a nonvanishing coupling from higher order terms involving explicit $SU(4)$ breaking. For example, from the technifermion mass terms we have

$$\Delta\mathcal{L}_{\text{eff}} \sim \frac{m_t/\sin\langle\theta\rangle}{\Lambda^{4-d}} (Qt^c)^\dagger_\alpha \text{tr}(\xi^T K \xi \Phi \xi^T P^\alpha \xi \Phi) + \text{h.c.} \quad (3.46)$$

This gives

$$g_{A\bar{t}t} \sim \frac{m_t}{f} \frac{\kappa + \tilde{\kappa}}{\Lambda^{4-d}} \sim \frac{N_c m_t^3}{16\pi^2 v^3} r \sin \theta \cos \theta, \quad (3.47)$$

where

$$r = \frac{\kappa + \tilde{\kappa}}{\kappa - \tilde{\kappa}}, \quad (3.48)$$

and we have used the vacuum condition Eq. (3.27) in the last step. More generally, there is a coupling to all standard model fermions given by

$$g_{A\bar{f}f} = Cr \frac{3m_t^2 m_f}{16\pi^2 v^3} \sin \theta \cos \theta, \quad (3.49)$$

where the normalization uncertainty is absorbed into an overall factor C that is the same for all fermions.

We can use these couplings to compute the A decays. The gauge boson decay rates are given by

$$\Gamma(A \rightarrow V_1 V_2) = \frac{g_{AV_1V_2}^2}{32\pi} \left[m_A^2 - (m_{V_1} + m_{V_2})^2 \right]^{3/2}, \quad (3.50)$$

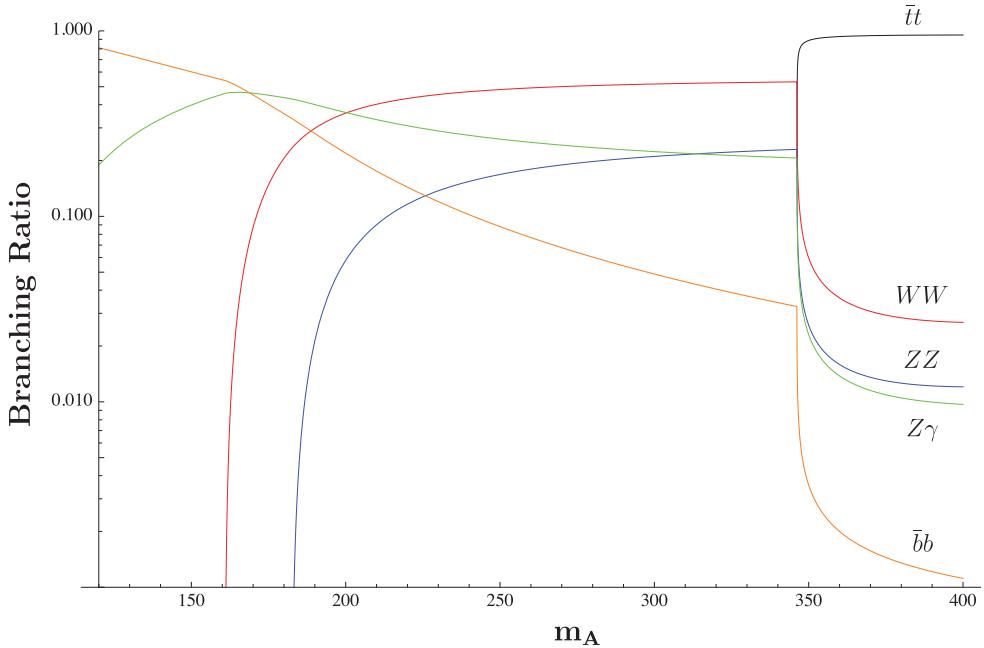


Fig. 2. Branching ratios for A decays assuming $Cr = 1$. We take $m_h = 120$ GeV, so the value of m_A fixes the value of the vacuum angle θ .

while the decay rate to fermions is

$$\Gamma(A \rightarrow \bar{f}f) = \frac{N_c g_{A\bar{q}q}^2}{8\pi} (m_A^2 - 4m_f^2)^{1/2}. \quad (3.51)$$

The branching ratios for A decays are shown in Fig. 2. It is important to remember that there is a large uncertainty in the overall normalization of the couplings to fermions. Nonetheless, it is clear that $A \rightarrow \bar{t}t$ dominates for $m_A > 2m_t$, while decays to gauge bosons dominate for smaller values of m_A down to $m_A \sim 150$ GeV.

We now briefly discuss the PNGB phenomenology, leaving detailed investigation for further work. The phenomenology of the composite Higgs is similar to the standard model, with suppressed couplings to the standard model particles. This has been studied in detail in Ref. [26]. The A phenomenology is more distinctive. Because the A couplings to gauge bosons and top quarks are significantly suppressed compared to the similar couplings of the standard model Higgs, direct production of A particles is tiny at the LHC. However, the theory is expected to contain resonances at the scale $\Lambda \sim \text{TeV}/\sin\theta$, some of which may have decays involving h and A particles. These may give a significant production rate. In particular, as shown in Ref. [11], we expect narrow spin-0 resonances at the scale Λ that can be efficiently produced via

gluon-gluon fusion through a top loop. The phenomenology of strong resonances in this model will be investigated in future work.

4 Precision Electroweak Corrections

We now discuss the precision electroweak constraints coming from the strong sector. The potentially large contributions are “oblique” corrections from electroweak gauge interactions, and a correction to $Z \rightarrow \bar{b}b$ due to the coupling of the top with the strong sector.

4.1 $Z \rightarrow \bar{b}b$

The coupling $g_{Z\bar{b}b}$ gets a non-universal contribution due to the top mass operator Eq. (2.6). There are contributions to $g_{Z\bar{b}b}$ from physics above the scale Λ arise from the diagrams shown in Fig. 3. The diagrams with two insertions of the top quark coupling are parameterized in the low-energy effective theory by terms of the form

$$\Delta\mathcal{L}_{\text{eff}} \sim \left(\frac{\Lambda}{\Lambda_t}\right)^{2(d-1)} Q_\alpha^\dagger \sigma^\mu Q^\beta \text{tr}(\Omega_\mu^\perp \xi^\dagger P_\beta^\dagger P_\gamma^\alpha \xi). \quad (4.1)$$

Eq. (4.1) gives a vanishing correction to $g_{Z\bar{b}b}$. This can be traced to the $Sp(4)$ custodial symmetry. Diagrams with four insertions of the top quark coupling give rise to operators such as

$$\Delta\mathcal{L}_{\text{eff}} \sim \left(\frac{\Lambda}{\Lambda_t}\right)^{4(d-1)} Q_\alpha^\dagger \sigma^\mu Q^\beta \text{tr}(\Omega_\mu^\perp \xi^\dagger P_\beta^\dagger P_\gamma^\dagger P_\gamma^\alpha \xi), \quad (4.2)$$

which give a tiny correction of order

$$\frac{\Delta g_{Z\bar{b}b}}{g_{Z\bar{b}b}} \sim \left(\frac{m_t}{4\pi v}\right)^4 \sin^2 \theta \sim 10^{-5} \sin^2 \theta. \quad (4.3)$$

PNGB loops do not contribute to $g_{Z\bar{b}b}$. Contributions involving standard model gauge bosons are included in the standard model contribution. We conclude that non-universal corrections to $Z \rightarrow \bar{b}b$ are small due to the $Sp(4)$ custodial symmetry.

4.2 T Parameter

The T parameter measures the breaking of custodial $SU(2)$. In this model, there is an enhanced $Sp(4)$ custodial symmetry that further suppresses the T parameter. The T parameter has potentially important contributions from the top quark coupling,

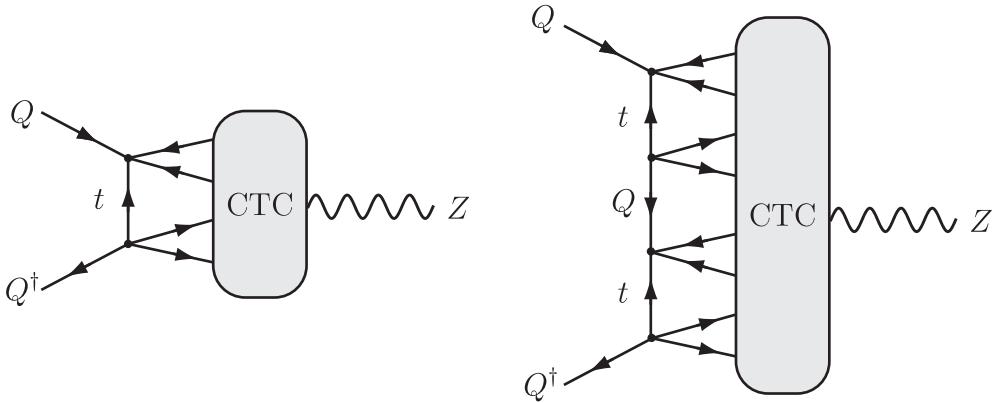


Fig. 3. Non-universal contributions to $Z \rightarrow \bar{b}b$ from the top quark coupling to the strong dynamics.

the $U(1)_Y$ gauge interactions, PNGB loops (including the composite Higgs), and 4-technifermion operators. We will discuss each of these contributions in turn.

We begin with the top quark coupling. The top quark contributes to the T parameter via a vacuum polarization involving a top quark loop. This contribution is UV finite, meaning it is insensitive to the physics at the scale Λ . This is included in the standard model fit to electroweak data. There are additional contributions to the T parameter from the top quark coupling Eq. (2.6) arising from physics above the scale Λ . In the effective theory below the scale Λ , the terms arising from two insertions of the top quark coupling that contribute to the gauge boson masses have the form

$$\begin{aligned} \Delta\mathcal{L}_{\text{eff}} \sim & \frac{\Lambda^2}{16\pi^2} \left(\frac{\Lambda}{\Lambda_t}\right)^{2(d-1)} \left[\text{tr}(\Omega^{\perp\mu}\Omega_\mu^\perp P_\alpha^\dagger P^\alpha) \right. \\ & + \text{tr}(\Omega^{\perp\mu}\Omega_\mu^\perp \Phi P^\alpha P_\alpha^\dagger \Phi^\dagger) \\ & \left. + \text{tr}(\Omega^{\perp\mu} P_\alpha^\dagger (\Omega_\mu^\perp)^T P^\alpha) \right]. \end{aligned} \quad (4.4)$$

The first two terms do not contribute to the T parameter because the custodial $SU(2)$ breaking is $\Delta I = 1$. The last term also gives a vanishing contribution to the T parameter due to the $Sp(4)$ custodial symmetry. There are nonvanishing contributions to the T parameter from four insertions of the top quark interaction, *e.g.*

$$\Delta\mathcal{L}_{\text{eff}} \sim \frac{\Lambda^2}{16\pi^2} \left(\frac{\Lambda}{\Lambda_t}\right)^{4(d-1)} \text{tr}(\Omega^{\perp\mu} P_\alpha^\dagger P^\beta \Omega_\mu^\perp P_\beta^\dagger P^\alpha). \quad (4.5)$$

The custodial symmetry violating mass term $\Delta m_W^2 = m_{W3}^2 - m_{W^\pm}^2$ is proportional to

$\sin^2 \theta$, so we obtain

$$\frac{\Delta m_W^2}{m_W^2} \sim \left(\frac{\Lambda}{\Lambda_t}\right)^{4(d-1)} \frac{g^2 f^2 \sin^2 \theta}{m_W^2} \sim \left(\frac{m_t}{4\pi v}\right)^4 \sim 10^{-5}. \quad (4.6)$$

This gives a negligible correction $\Delta T = \alpha^{-1} \Delta m_W^2 / m_W^2 \sim 10^{-3}$.

We now discuss the contribution to the T parameter from $U(1)_Y$ loops. This arises from

$$\Delta \mathcal{L}_{\text{eff}} \sim \frac{g'^2 f^2}{16\pi^2} \text{tr}(\Omega^{\perp\mu} \xi^\dagger Y \xi \Omega_\mu^\perp \xi^\dagger Y \xi), \quad (4.7)$$

where

$$Y = Y^\dagger = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{1}{2}\tau_3 \end{pmatrix} \quad (4.8)$$

is the $U(1)_Y$ generator. The custodial symmetry violating mass term is proportional to $\sin^4 \theta$, and we obtain

$$\frac{\Delta m_W^2}{m_W^2} \sim \frac{g'^2}{16\pi^2} \frac{g^2 f^2 \sin^4 \theta}{m_W^2} \sim \frac{g'^2}{16\pi^2} \sin^2 \theta \sim 10^{-3} \sin^2 \theta. \quad (4.9)$$

We see that this contribution is suppressed at small $\sin \theta$.

There are also contributions from below the scale Λ due to PNGB loops. The pseudoscalar does not contribute, while the Higgs contributes

$$\begin{aligned} \Delta T_{\text{IR}} &= -\frac{3}{8\pi \cos^2 \theta_W} \left[\cos^2 \theta \ln \frac{m_h}{\Lambda} - \ln \frac{m_{h,\text{ref}}}{\Lambda} \right] \\ &= -\frac{3}{8\pi \cos^2 \theta_W} \left[\ln \frac{m_h}{m_{h,\text{ref}}} - \sin^2 \theta \ln \frac{m_h}{\Lambda} \right], \end{aligned} \quad (4.10)$$

where we have included the subtraction due to the reference Higgs mass. The first term in Eq. (4.10) is the usual standard model contribution to the T parameter from the Higgs loop. The logarithmic dependence on Λ in the second term in Eq. (4.10) represents a logarithmic UV divergence in the effective theory below the scale Λ . The cutoff dependence is canceled by the Λ dependence of the effective couplings in Eq. (4.7).

4.3 S Parameter

The S parameter gets corrections from all sources of electroweak symmetry breaking. The contribution from physics above the scale Λ are parameterized by the following 4-derivative terms in the effective Lagrangian below Λ :

$$\Delta \mathcal{L}_S = -\frac{1}{4} c_F \text{tr}(\mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu}) - \frac{1}{2} c_D \text{tr}(\mathcal{D}^{\mu\nu} \mathcal{D}_{\mu\nu}) \quad (4.11)$$

where

$$\mathcal{F}_{\mu\nu} = -i[\nabla_\mu, \nabla_\nu], \quad (4.12)$$

$$\mathcal{D}_{\mu\nu} = \nabla_\mu \Omega_\nu^\perp, \quad (4.13)$$

where the covariant derivative is defined in Appendix C.¹ These contribute to the S parameter

$$S_{UV} = 16\pi(\frac{1}{2}c_{\mathcal{F}} - c_{\mathcal{D}}) \sin^2 \theta. \quad (4.14)$$

This represents the contribution to the S parameter from physics above the scale Λ .

There are also contributions to the S parameter from physics below the scale Λ , namely the PNGBs. The pseudoscalar A does not contribute to the S parameter because it has only “tadpole” couplings to electroweak gauge bosons, and therefore does not give a momentum-dependent contribution. The composite Higgs loop gives a contribution

$$\begin{aligned} S_{IR} &= \frac{1}{6\pi} \left[\cos^2 \theta \ln \frac{m_h}{\Lambda} - \ln \frac{m_{h,ref}}{\Lambda} \right] \\ &= \frac{1}{6\pi} \left[\ln \frac{m_h}{m_{h,ref}} - \sin^2 \theta \ln \frac{m_h}{4\pi v / \sin \theta} \right], \end{aligned} \quad (4.15)$$

where $m_{h,ref}$ is the reference Higgs mass. The first term in Eq. (4.15) is the usual standard model contribution to the S parameter from the Higgs loop, and the logarithmic dependence on Λ in the second term is canceled by the Λ dependence of the counterterms Eq. (4.11).

4.4 Electroweak Fit

We now put the results above together to discuss the fit to precision electroweak data. One important parameter is m_h , the mass of the scalar PNGB. Recall that m_h is independent of θ . For $\sin \theta \ll 1$, the couplings of h become those of a composite Higgs boson, and the precision electroweak fit reduces to the standard model with Higgs mass m_h . A good electroweak fit therefore requires $m_h^2 \lesssim 180$ GeV. Smaller values of m_h^2 allow larger values of $\sin \theta$ to get a good electroweak fit. This is illustrated in Fig. 4. This assumes $m_h = 120$ GeV, and that the UV contribution to the S parameter (see Eq. (4.14)) is positive and given by the QCD value [3], multiplied

¹Because of the relations Eq. (B.22) among the generators, we do not get any new invariants by contracting $Sp(4)$ indices using the $Sp(4)$ invariant metric Φ .

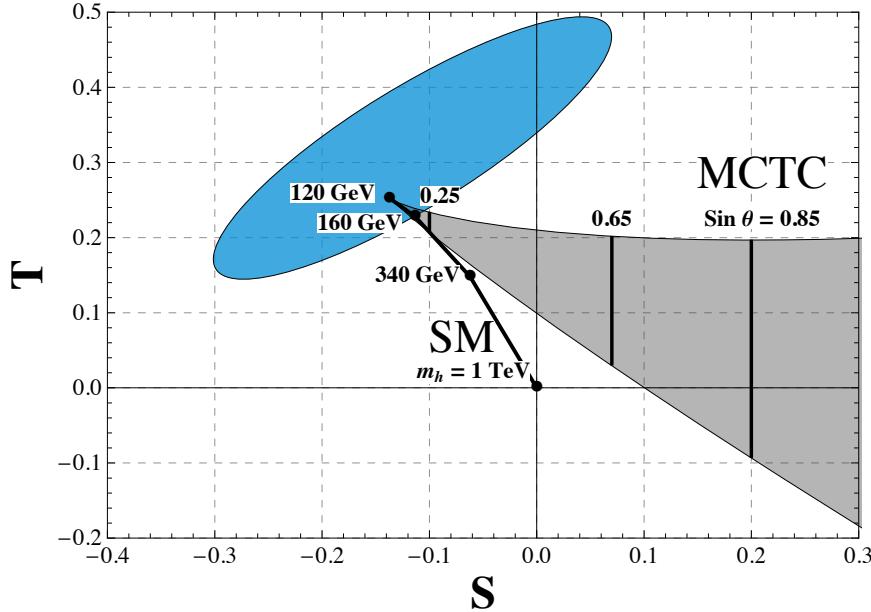


Fig. 4. Precision electroweak fit in the model described in the text for $m_h = 120$ GeV.

by $2/3$ to extrapolate from $N_c = 3$ to $N_c = 2$. We use the recent electroweak fit of Ref. [27]. Like the standard model, the present model has a single parameter (in this case $\sin \theta$) that controls the precision electroweak fit, and has a good fit for a small range of this parameter.

However, the limit $\theta \ll 1$ is fine tuned, and we must be close to this limit to get a good electroweak fit. To quantify this tuning, we evaluate the sensitivity of the electroweak VEV to the technifermion mass κ , a parameter in the fundamental theory that controls the vacuum angle θ . We have

$$\text{sensitivity} = \frac{d \ln v^2}{d \ln \kappa} = -\frac{2}{\tan^2 \theta}. \quad (4.16)$$

As expected, this goes as $f^2/v^2 \sim \theta^{-2}$ for small θ . For $\theta \sim 0.25$ the sensitivity is ~ -30 . The fine tuning is further reduced for smaller m_h . Fine tuning may be completely absent if there are additional positive contributions to the T parameter. In this case, we can allow $\sin \theta \lesssim 0.5$, which gives a sensitivity parameter ~ 5 .

5 Conclusions

We have analyzed the minimal theory of conformal technicolor, an $SU(2)$ gauge theory with fundamentals. This gives a plausible theory of strong electroweak symmetry breaking, including the possibility of decoupling flavor physics to high scales. We have analyzed the vacuum structure of this theory and showed that it naturally gives rise to a vacuum with a composite Higgs boson. We analyzed the constraints from precision electroweak data and showed that a good electroweak fit can be obtained with very mild fine tuning, even if the strong contribution to the S parameter is positive and unsuppressed. The minimal model of conformal technicolor may therefore sole the two main problems of strong electroweak symmetry breaking.

The characteristic phenomenological features of this model are a composite Higgs h with mass close to the LEP bound, and a heavier pseudoscalar A decaying to $t\bar{t}$ or WW , ZZ , and $Z\gamma$ (but not $\gamma\gamma$). The compositeness of the Higgs results in reduced h couplings to the standard model fields, as in all composite Higgs models. Direct A production is small at the LHC, but there may be observable effects due to strongly-coupled heavy resonances that decay to A (and/or h). In particular, the coupling of the top to the strong breaking sector may allow may allow production of narrow spin-0 resonances [11]. The phenomenology of this model will be studied in detail in future work.

Another area in which more work is needed is the flavor physics. The flavor scale can be pushed to high scales if the dimension d of the technifermion bilinear is small, so the present model can plausibly accommodate flavor. However, it is important to have an explicit model for flavor at high scales. Such a model will show that the flavor problem that has been postponed to higher scales by large anomalous dimensions is tractable. An explicit flavor model is also necessary to to check that flavor-changing neutral currents are truly suppressed, and to determine the lowest allowed value of the flavor scale. This question will also be addressed in future work.

Acknowledgments

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Appendix A: The Space of Vacua

In this appendix, we derive the most general form of the fermion condensate

$$\langle \Psi^a \Psi^b \rangle \propto \Phi^{ab}, \quad (\text{A.1})$$

where

$$\Phi^T = -\Phi. \quad (\text{A.2})$$

We define the phase of Φ by choosing the constant of proportionality in Eq. (A.1) to be real.

The space of vacua of this theory is given by the space of possible VEVs of gauge invariant operators such as Eq. (A.1). In particular, any VEVs related by symmetries of the fundamental theory will correspond to states with degenerate energy. We assume that it is sufficient to study symmetry transformations of the fermion condensate to identify the space of vacua. Global $SU(4)$ transformations act as

$$\Phi \mapsto U\Phi U^T. \quad (\text{A.3})$$

Note in particular the transformation

$$\Psi \mapsto i\Psi, \quad (\text{A.4})$$

which maps $\Phi \mapsto -\Phi$. This is a Z_2 transformation because the transformation $\Psi \mapsto -\Psi$ is a gauge transformation. We also have CP transformations, under which

$$\Phi \mapsto \Phi^\dagger. \quad (\text{A.5})$$

The Pfaffian

$$\text{Pf}(\Phi) = \frac{1}{4}\epsilon_{abcd}\Phi^{ab}\Phi^{cd} \quad (\text{A.6})$$

is left invariant by $SU(4)$ transformations, while under CP it transforms as $\text{Pf}(\Phi) \mapsto \text{Pf}(\Phi)^*$. We assume that the theory has CP preserving vacua, which implies that $\text{Pf}(\Phi)$ is real. We can then choose the normalization of the condensate so that

$$\text{Pf}(\Phi) = \pm 1. \quad (\text{A.7})$$

Note that there is no symmetry of the theory that can change the Pfaffian, so there is no reason that states with opposite sign for $\text{Pf}(\Phi)$ are degenerate in energy. The

natural assumption is therefore that the physical vacua correspond to one or the other sign. We will refer to the cases $\text{Pf}(\Phi) = \pm 1$ as “+” and “−” vacua, respectively.

We now assume that the vacuum preserves $Sp(4) \in SU(4)$. This is satisfied if we impose

$$\Phi^\dagger \Phi = 1. \quad (\text{A.8})$$

To see this, it is sufficient to note that this is satisfied by canonical $Sp(4)$ condensates such as

$$\Phi_0 = \begin{pmatrix} 0 & 1_2 \\ -1_2 & 0 \end{pmatrix}, \quad (\text{A.9})$$

and that this condition is invariant under $SU(4)$. Indeed, the defining relations Eqs. (A.2), (A.7), and (A.8) are all invariant under $SU(4)$ and CP transformations. Solving these constraints will therefore give us the most general condensate Φ . The most general antisymmetric matrix can be written

$$\Phi = \begin{pmatrix} a\epsilon & c \\ -c^T & b\epsilon \end{pmatrix}. \quad (\text{A.10})$$

where A and b are complex and c is a 2×2 complex matrix. Eq. (A.8) implies

$$cc^\dagger + |a|^2 1_2 = c^\dagger c + |b|^2 1_2 = 1_2, \quad (\text{A.11})$$

$$ac^\dagger \epsilon + b^* \epsilon c^T = 0. \quad (\text{A.12})$$

Eq. (A.11) implies

$$c = ru \quad (\text{A.13})$$

where u is unitary and r is a real number given by

$$r = 1 - |a|^2 = 1 - |b|^2. \quad (\text{A.14})$$

We can use a $SU(2)_W$ transformation to set $u = e^{i\gamma}$, where γ is real. Then Eq. (A.12) implies

$$ae^{i\gamma} = -(be^{i\gamma})^*. \quad (\text{A.15})$$

We conclude that up to $SU(2)_W$ transformations the most general condensate satisfying Eqs. (A.2) and (A.8) is

$$\Phi = e^{i\gamma} \begin{pmatrix} e^{i\alpha} \cos \theta \epsilon & \sin \theta 1_2 \\ -\sin \theta 1_2 & -e^{-i\alpha} \cos \theta \epsilon \end{pmatrix}. \quad (\text{A.16})$$

This has $\text{Pf}(\Phi) = -e^{2i\gamma}$, so the most general vacuum is either Φ or $i\Phi$, where

$$\Phi = \begin{pmatrix} e^{i\alpha} \cos \theta \epsilon & \sin \theta 1_2 \\ -\sin \theta 1_2 & -e^{-i\alpha} \cos \theta \epsilon \end{pmatrix}. \quad (\text{A.17})$$

We can change the sign of the block off-diagonal entry with a $SU(2)_W$ (or $U(1)_Y$) gauge transformation, so the physically distinct vacua are labelled by θ in the range

$$0 \leq \theta \leq \pi. \quad (\text{A.18})$$

Note that the vacua $\theta = 0$ and $\theta = \pi$ are related by the Z_2 transformation $\Psi \mapsto i\Psi$.

Appendix B: Generators

This appendix derives the $Sp(4)$ and $SU(4)/Sp(4)$ generators associated with a general $Sp(4)$ metric Eq. (A.17). We can write an arbitrary $SU(N)$ generator as

$$T = T_{\parallel} + T_{\perp}, \quad (\text{B.19})$$

where

$$T_{\parallel} = \frac{1}{2}(T - \Phi T^T \Phi^\dagger), \quad (\text{B.20})$$

$$T_{\perp} = \frac{1}{2}(T + \Phi T^T \Phi^\dagger). \quad (\text{B.21})$$

This is a projection in the sense that $(T_{\parallel})_{\parallel} = T_{\parallel}$, $(T_{\parallel})_{\perp} = 0$, etc. These generators satisfy

$$T_{\parallel}\Phi + \Phi T_{\parallel}^T = 0, \quad (\text{B.22})$$

$$T_{\perp}\Phi - \Phi T_{\perp}^T = 0. \quad (\text{B.23})$$

This means that the T_{\parallel} are $Sp(N)$ generators. We identify T_{\perp} with the broken generators of $SU(N)/Sp(N)$. These definitions imply $[T_{\parallel}, T_{\perp}] = \sum T_{\perp}$, i.e. the broken generators form a linear representation of $Sp(N)$. Also, the broken and unbroken generators are orthogonal in the sense that $\text{tr}(T_{\parallel} T_{\perp}) = 0$.

To work out the explicit form of the generators it is useful to start in the electroweak vacuum

$$\Phi_0 = \begin{pmatrix} \epsilon & 0 \\ 0 & -\epsilon \end{pmatrix} \quad (\text{B.24})$$

and perform a $SU(4)$ rotation to the general vacuum:

$$\Phi = U_0 \Phi_0 U_0^T = \begin{pmatrix} \cos \theta \epsilon & \sin \theta 1_2 \\ -\sin \theta 1_2 & -\cos \theta \epsilon \end{pmatrix}, \quad (\text{B.25})$$

where

$$U_0 = \begin{pmatrix} \cos \frac{\theta}{2} \mathbf{1}_2 & \sin \frac{\theta}{2} \epsilon \\ \sin \frac{\theta}{2} \epsilon & \cos \frac{\theta}{2} \mathbf{1}_2 \end{pmatrix} \in SU(4). \quad (\text{B.26})$$

We can work out the generators for the vacuum Eq. (B.24) and rotate them into the general basis. In the basis Eq. (B.24) the generators are

$$T_{\parallel,0} = \begin{pmatrix} a_i \sigma_i & c_0 + i c_i \sigma_i \\ c_0 - i c_i \sigma_i & b_i \sigma_i \end{pmatrix}, \quad T_{\perp,0} = \begin{pmatrix} x \mathbf{1}_2 & i y_0 + y_i \sigma_i \\ -i y_0 + y_i \sigma_i & -x \mathbf{1}_2 \end{pmatrix}, \quad (\text{B.27})$$

where a_i , b_i , c_0 , c_i , x , y_0 , and y_i are real. Performing the transformation

$$T = U_0 T_0 U_0^\dagger, \quad (\text{B.28})$$

we find bases in a general vacuum state given by

$$\begin{aligned} T_{\parallel}^1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T_{\parallel}^2 = \begin{pmatrix} 0 & i \sigma_1 \\ -i \sigma_1 & 0 \end{pmatrix}, \quad T_{\parallel}^3 = \begin{pmatrix} s \mathbf{1}_2 & c \epsilon \\ -c \epsilon & -s \mathbf{1}_2 \end{pmatrix}, \\ T_{\parallel}^4 &= \begin{pmatrix} 0 & i \sigma_3 \\ -i \sigma_3 & 0 \end{pmatrix}, \quad T_{\parallel}^5 = \begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix}, \quad T_{\parallel}^6 = \begin{pmatrix} c \sigma_1 & s \sigma_3 \\ s \sigma_3 & c \sigma_1 \end{pmatrix}, \\ T_{\parallel}^7 &= \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \quad T_{\parallel}^8 = \begin{pmatrix} c \sigma_3 & -s \sigma_1 \\ -s \sigma_1 & c \sigma_3 \end{pmatrix}, \\ T_{\parallel}^9 &= \begin{pmatrix} c \sigma_2 & -i s \mathbf{1}_2 \\ i s \mathbf{1}_2 & -c \sigma_2 \end{pmatrix}, \quad T_{\parallel}^{10} = \begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix}, \end{aligned} \quad (\text{B.29})$$

and

$$\begin{aligned} T_{\perp}^1 &= \begin{pmatrix} s \sigma_1 & -c \sigma_3 \\ -c \sigma_3 & s \sigma_1 \end{pmatrix}, \quad T_{\perp}^2 = \begin{pmatrix} s \sigma_2 & i c \mathbf{1}_2 \\ -i c \mathbf{1}_2 & -s \sigma_2 \end{pmatrix}, \quad T_{\perp}^3 = \begin{pmatrix} s \sigma_3 & c \sigma_1 \\ c \sigma_1 & s \sigma_3 \end{pmatrix}, \\ T_{\perp}^4 &= \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \quad T_{\perp}^5 = \begin{pmatrix} c \mathbf{1}_2 & -s \epsilon \\ s \epsilon & -c \mathbf{1}_2 \end{pmatrix}, \end{aligned} \quad (\text{B.30})$$

where $s = \sin \theta$, $c = \cos \theta$. The transformation Eq. (B.28) preserves trace orthogonality, and it is easy to see that these generators are orthogonal. In the technicolor limit $\theta \rightarrow \frac{\pi}{2}$ we have

$$T_{\perp}^i \rightarrow \begin{pmatrix} \sigma_i & 0 \\ 0 & \sigma_i^T \end{pmatrix}, \quad i = 1, 2, 3, \quad (\text{B.31})$$

and we recognize these as the generators corresponding to the longitudinal components of the W and Z . The physical PNGBs therefore correspond to the generators $T_{\perp}^{4,5}$. Under the preserved CP symmetries, the PNGB fields transform as $\Pi \mapsto -\Pi^*$ (see Eq. (C.47) below) so the PNGB corresponding to T_{\perp}^4 (T_{\perp}^5) is even (odd). We therefore identify T_{\perp}^4 (T_{\perp}^5) with the generator corresponding to h (A).

Appendix C: Nonlinear Realization

In this appendix, we review some details of the nonlinear realization of $SU(4)/Sp(4)$. The Nambu-Goldstone bosons can be described as broken symmetry excitations of the order parameter Φ :

$$\Phi \rightarrow \xi \Phi \xi^T, \quad (\text{C.32})$$

where

$$\xi = e^{i\Pi}, \quad (\text{C.33})$$

with Π a linear combination of the broken generators T_\perp defined above. The condensate Φ is $Sp(4)$ invariant:

$$\Phi = V\Phi V^T \quad (\text{C.34})$$

for all $V \in Sp(4)$. We therefore define the transformation under $U \in SU(4)$

$$\xi \Phi \xi^T \mapsto U \xi \Phi \xi^T U^T = \xi' \Phi \xi'^T, \quad (\text{C.35})$$

where [28]

$$\xi \mapsto \xi' = U \cdot \xi \cdot V^\dagger(U, \xi). \quad (\text{C.36})$$

Here $V(U, \xi) \in Sp(4)$ is defined by the condition that Π' is a linear combination of broken generators. Note that V depends on x through ξ .

We now construct the $SU(4)$ invariant terms in the effective Lagrangian, following the standard construction of Ref. [28]. We define the semi-covariant derivative

$$D_\mu \xi = \partial_\mu \xi - i \mathcal{A}_\mu \xi, \quad (\text{C.37})$$

where \mathcal{A}_μ are gauge fields for $SU(4)$. This notation is appropriate for weakly gauging all of $SU(4)$, although we will only gauge the $SU(2)_W \times U(1)_Y$ subgroup. These fields do not transform covariantly under $SU(4)$ gauge transformations, but the quantity

$$\Omega_\mu = i \xi^\dagger D_\mu \xi \quad (\text{C.38})$$

transforms like a $Sp(4)$ gauge field:

$$\Omega_\mu \mapsto V(\Omega_\mu + i \partial_\mu) V^\dagger. \quad (\text{C.39})$$

We project Ω_μ onto fields parallel and perpendicular to the unbroken $Sp(4)$ direction using Eq. (B.20):

$$\Omega_\mu^{\parallel} = \frac{1}{2}(\Omega_\mu - \Phi\Omega_\mu^T\Phi^\dagger), \quad (\text{C.40})$$

$$\Omega_\mu^{\perp} = \frac{1}{2}(\Omega_\mu + \Phi\Omega_\mu^T\Phi^\dagger). \quad (\text{C.41})$$

These then transform as

$$\Omega_\mu^{\parallel} \mapsto V(\Omega_\mu^{\parallel} + i\partial_\mu)V^\dagger, \quad (\text{C.42})$$

$$\Omega_\mu^{\perp} \mapsto V\Omega_\mu^{\perp}V^\dagger. \quad (\text{C.43})$$

We therefore define the $Sp(4)$ covariant derivative

$$\nabla_\mu = \partial_\mu - i\Omega_\mu^{\parallel} \quad (\text{C.44})$$

for fields transforming under $Sp(4)$. For example, we can define

$$\mathcal{D}_{\mu\nu} = \nabla_\mu\Omega_\nu^{\perp} = \partial_\mu\Omega_\nu^{\perp} - i[\Omega_\mu^{\parallel}, \Omega_\nu^{\perp}] \mapsto V\mathcal{D}_{\mu\nu}V^\dagger. \quad (\text{C.45})$$

We now discuss CP invariance. Under CP the condensate transforms as $\Phi \mapsto \Phi^\dagger$. The condensate Eq. (A.17) leaves invariant the combination of this transformation and the discrete symmetry $\Psi \mapsto i\Psi$, which maps $\Phi \mapsto -\Phi = \Phi^T$. The preserved CP transformation is therefore

$$CP : \Phi \mapsto \Phi^*. \quad (\text{C.46})$$

Under this symmetry we have $\xi \mapsto \xi^*$, and therefore

$$CP : \Pi(x) \mapsto -\Pi^*(x^P), \quad (\text{C.47})$$

where x^P is the parity transformed spacetime point. If we impose the standard CP transformation on the gauge fields

$$CP : A_\mu(x) \mapsto -A_\mu^*(x^P) \quad (\text{C.48})$$

we have the CP transformations

$$\Omega_\mu^{\perp}(x) \mapsto -[\Omega_\mu^{\perp}(x^P)]^*, \quad \Omega_\mu^{\parallel}(x) \mapsto -[\Omega_\mu^{\parallel}(x^P)]^*. \quad (\text{C.49})$$

References

- [1] M. A. Luty and T. Okui, JHEP **0609**, 070 (2006) [arXiv:hep-ph/0409274].
- [2] M. A. Luty, JHEP **0904**, 050 (2009) [arXiv:0806.1235 [hep-ph]].
- [3] B. Holdom and J. Terning, Phys. Lett. B **247**, 88 (1990); M. Golden and L. Randall, Nucl. Phys. B **361**, 3 (1991); M. E. Peskin and T. Takeuchi, Phys. Rev. D **46**, 381 (1992).
- [4] A. Manohar and H. Georgi, Nucl. Phys. B **234**, 189 (1984).
- [5] C. Csaki, C. Grojean, L. Pilo and J. Terning, Phys. Rev. Lett. **92**, 101802 (2004) [arXiv:hep-ph/0308038].
- [6] H. Davoudiasl, J. L. Hewett, B. Lillie and T. G. Rizzo, Phys. Rev. D **70**, 015006 (2004) [arXiv:hep-ph/0312193]; G. Burdman and Y. Nomura, Phys. Rev. D **69**, 115013 (2004) [arXiv:hep-ph/0312247]; R. Barbieri, A. Pomarol, R. Rattazzi and A. Strumia, Nucl. Phys. B **703**, 127 (2004) [arXiv:hep-ph/0405040]; K. Agashe, C. Csaki, C. Grojean and M. Reece, JHEP **0712**, 003 (2007) [arXiv:0704.1821 [hep-ph]].
- [7] D. B. Kaplan and H. Georgi, Phys. Lett. B **136**, 183 (1984); D. B. Kaplan, H. Georgi and S. Dimopoulos, Phys. Lett. B **136**, 187 (1984); H. Georgi and D. B. Kaplan, Phys. Lett. B **145**, 216 (1984); M. J. Dugan, H. Georgi and D. B. Kaplan, Nucl. Phys. B **254**, 299 (1985).
- [8] R. Contino, Y. Nomura and A. Pomarol, Nucl. Phys. B **671**, 148 (2003) [arXiv:hep-ph/0306259]; K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B **719**, 165 (2005) [arXiv:hep-ph/0412089].
- [9] E. Katz, A. E. Nelson and D. G. E. Walker, JHEP **0508**, 074 (2005) [arXiv:hep-ph/0504252].
- [10] B. Gripaios, A. Pomarol, F. Riva and J. Serra, JHEP **0904**, 070 (2009) [arXiv:0902.1483 [hep-ph]].
- [11] J. A. Evans and M. A. Luty, Phys. Rev. Lett. **103**, 101801 (2009) [arXiv:0904.2182 [hep-ph]].
- [12] L. Susskind, Phys. Rev. D **20**, 2619 (1979); S. Weinberg, Phys. Rev. D **13**, 974 (1976); S. Weinberg, Phys. Rev. D **19**, 1277 (1979).

- [13] Y. Iwasaki, K. Kanaya, S. Kaya, S. Sakai and T. Yoshie, Phys. Rev. D **69**, 014507 (2004) [arXiv:hep-lat/0309159]; J. Braun and H. Gies, Phys. Lett. B **645**, 53 (2007) [arXiv:hep-ph/0512085]; T. Appelquist, G. T. Fleming and E. T. Neil, Phys. Rev. Lett. **100**, 171607 (2008) [Erratum-ibid. **102**, 149902 (2009)] [arXiv:0712.0609 [hep-ph]]; Phys. Rev. D **79**, 076010 (2009) [arXiv:0901.3766 [hep-ph]]; A. Deuzeman, M. P. Lombardo and E. Pallante, Phys. Lett. B **670**, 41 (2008) [arXiv:0804.2905 [hep-lat]]; arXiv:0904.4662 [hep-ph]; arXiv:0911.2207 [hep-lat]; Z. Fodor, K. Holland, J. Kuti, D. Nogradi and C. Schroeder, arXiv:0911.2463 [hep-lat].
- [14] T. Appelquist, A. Ratnaweera, J. Terning and L. C. R. Wijewardhana, Phys. Rev. D **58**, 105017 (1998) [arXiv:hep-ph/9806472].
- [15] T. Banks and A. Zaks, Nucl. Phys. B **196**, 189 (1982).
- [16] T. Appelquist, M. Einhorn, T. Takeuchi and L. C. R. Wijewardhana, Phys. Lett. B **220**, 223 (1989); R. S. Chivukula, A. G. Cohen and K. D. Lane, Nucl. Phys. B **343**, 554 (1990); C. T. Hill, Phys. Lett. B **266**, 419 (1991); Phys. Lett. B **345**, 483 (1995) [arXiv:hep-ph/9411426].
- [17] B. Holdom, Phys. Lett. B **150**, 301 (1985); T. W. Appelquist, D. Karabali and L. C. R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986); K. Yamawaki, M. Bando and K. i. Matumoto, Phys. Rev. Lett. **56**, 1335 (1986); T. Appelquist and L. C. R. Wijewardhana, Phys. Rev. D **35**, 774 (1987); Phys. Rev. D **36**, 568 (1987).
- [18] S. Samuel, Nucl. Phys. B **347**, 625 (1990); M. Dine, A. Kagan and S. Samuel, Phys. Lett. B **243**, 250 (1990).
- [19] E. H. Simmons, Nucl. Phys. B **312**, 253 (1989); C. D. Carone and E. H. Simmons, Nucl. Phys. B **397**, 591 (1993) [arXiv:hep-ph/9207273].
- [20] T. DeGrand, arXiv:0910.3072 [hep-lat].
- [21] G. Mack, Commun. Math. Phys. **55**, 1 (1977); B. Grinstein, K. A. Intriligator and I. Z. Rothstein, Phys. Lett. B **662**, 367 (2008) [arXiv:0801.1140 [hep-ph]].
- [22] R. Rattazzi, V. S. Rychkov, E. Tonni and A. Vichi, JHEP **0812**, 031 (2008) [arXiv:0807.0004 [hep-th]].
- [23] V. S. Rychkov and A. Vichi, Phys. Rev. D **80**, 045006 (2009) [arXiv:0905.2211 [hep-th]].

- [24] M. E. Peskin, Nucl. Phys. B **175**, 197 (1980); J. Preskill, Nucl. Phys. B **177**, 21 (1981).
- [25] M. Schmaltz and D. Tucker-Smith, Ann. Rev. Nucl. Part. Sci. **55**, 229 (2005) [arXiv:hep-ph/0502182].
- [26] G. F. Giudice, C. Grojean, A. Pomarol and R. Rattazzi, JHEP **0706**, 045 (2007) [arXiv:hep-ph/0703164].
- [27] J. Erler and P. Langacker, Phys. Rev. Lett. **105**, 031801 (2010) [arXiv:1003.3211 [hep-ph]].
- [28] S. R. Coleman, J. Wess and B. Zumino, Phys. Rev. **177**, 2239 (1969).